

# Pulsed Electric Propulsion Thrust Stand Calibration Method

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## I. Abstract

THE evaluation of the performance of any propulsion device requires the accurate measurement of thrust. While chemical rocket thrust is typically measured using a load cell, the low thrust levels associated with electric propulsion (EP) systems necessitate the use of much more sensitive measurement techniques. The design and development of electric propulsion thrust stands that employ a conventional hanging pendulum arm connected to a balance mechanism consisting of a secondary arm and variable linkage have been reported in recent publications by Polzin *et al.*<sup>1,2</sup> These works focused on performing steady-state thrust measurements and employed a static analysis of the thrust stand response.

In the present work, we present a calibration method and data that will permit pulsed thrust measurements using the Variable Amplitude Hanging Pendulum with Extended Range (VAHPER)<sup>1</sup> thrust stand. Pulsed thrust measurements are challenging in general because the pulsed thrust (impulse bit) occurs over a short timescale (typically 1  $\mu$ s to 1 ms) and cannot be resolved directly. Consequently, the imparted impulse bit must be inferred through observation of the change in thrust stand motion effected by the pulse.<sup>3,4</sup>

Pulsed thrust measurements have typically only consisted of single-shot operation. In the present work, we discuss repetition-rate pulsed thruster operation and describe a method to perform these measurements. The thrust stand response can be modeled as a spring-mass-damper system with a repetitive delta forcing function to represent the impulsive action of the thruster.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \sum_{i=0}^n \frac{I_{\text{bit}}}{m_{\text{eff}}} \delta(t - i/f) \quad (1)$$

where  $\omega_n$  is the natural frequency,  $\zeta$  is the damping coefficient,  $m_{\text{eff}}$  is the effective mass of the system, and  $f$  is the pulse frequency. This differential equation has a closed form solution that changes depending on the value of  $\zeta$ . Solving under the initial conditions  $x(0) = \dot{x}(0) = 0$  yields

$$x(t) = \begin{cases} \sum_{i=0}^n \frac{I_{\text{bit}}}{2m_{\text{eff}}\omega_n\sqrt{\zeta^2-1}} e^{-\omega_n\zeta(t-i/f)} \left[ e^{\omega_n\sqrt{\zeta^2-1}(t-i/f)} - e^{-\omega_n\sqrt{\zeta^2-1}(t-i/f)} \right] H(t-i/f), & \text{for } \zeta > 1, \\ \sum_{i=0}^n \frac{I_{\text{bit}}}{m_{\text{eff}}} (t-i/f) e^{-\omega_n(t-i/f)} H(t-i/f), & \text{for } \zeta = 1, \\ \sum_{i=0}^n \frac{I_{\text{bit}}}{m_{\text{eff}}\omega_n\sqrt{1-\zeta^2}} e^{-\omega_n\zeta(t-i/f)} \sin\left(\omega_n\sqrt{1-\zeta^2}(t-i/f)\right) H(t-i/f), & \text{for } \zeta < 1, \end{cases} \quad (2)$$

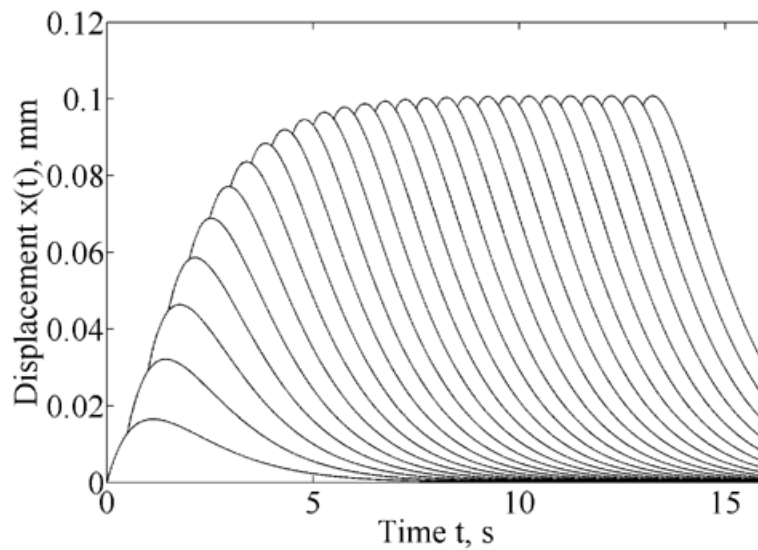
where  $H$  is the Heaviside function. Equation (2) is plotted in Fig. 1 for  $\zeta = 1$  and  $n$  ranging from 1 to 27. These data show that the solution asymptotes to one that oscillates about a given displacement after a finite number of pulses until the impulsive forcing function is removed from the system after the  $n^{\text{th}}$  pulse. In addition we observe that the response of the thrust stand occurs on the order of seconds, which is much greater than the known electric thruster pulse width, validating the modeling assumption of impulsive force application.

The apparatus shown schematically in Fig. 2 was fabricated to permit the application of a regular, impulsive, repetitive force in a non-contact manner. As the solenoid is energized it exerts a force on the magnet, which is

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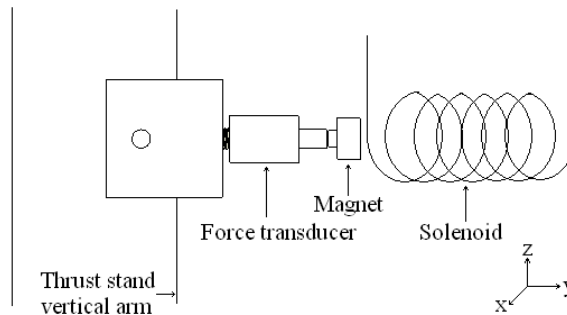
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**Figure 1.** Computed thrust stand displacement (using Eq. (2)) for  $\zeta = 1$  showing the response for the application of repetitive impulse bits ranging from  $n$  of 1 to 27.

physically connected to the thrust stand pendulum arm. This force can be measured using a standard piezoelectric force transducer. The measurement of the applied impulse bit and the induced thrust stand motion permit the determination of the coefficients in Eq. 1), which can then be used to find the unknown thruster impulse bit given a measured thrust stand response.



**Figure 2.** Schematic view of the pulsed thrust calibration apparatus.

## References

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